

OFF-SHELL BRST-VSUSY SUPERALGEBRA FOR D=4 BF THEORIES IN THE SUPERCONNECTION FORMALISM

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Abstract

We propose the superconnection formalism to construct the off-shell BRST-VSUSY superalgebra for D=4 BF theories. The method is based on the natural introduction of physical fields as well as auxiliary fields via superconnections and their associated supercurvatures defined on a superspace. We also give a prescription to build the off-shell BRST-VSUSY exact quantum action.

Keywords: Superconnection; off-shell BRST-VSUSY superalgebra and BF theories.

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It is known that the BRST transformations in Yang-Mills (YM) theories can be derived systematically from the horizontality conditions imposed on the supercurvature associated with a superconnection defined on a (4,2)-dimensional superspace, the so-called BRST superspace, obtained by extending spacetime with two ordinary anticommuting coordinates [1]. In such BRST superspace formalism, the gauge fields, the ghost and antighost fields are interpreted as the lowest components of the superfield components of the superconnection. This treatment is also considered in the description for tensor gauge fields [2].

In Ref.[3], the authors study the topological YM theory in the superconnection framework. In such way the horizontality conditions are modified in order to obtain the BRST transformation rules of the component fields in a systematic way.

However, the quantization of BF theories in four dimension has been discussed within the superfibre bundle formalism [4]. In this geometrical framework, the fields present in the quantized theory have been described through a superconnections introduced over a principal superfibre bundle. We note that this description is also considered for the so-called BF-YM theory [5].

In Ref. [6], the author developed the BRST superspace formalism in order to perform the quantization of BF theories as a model with reducible gauge symmetry, see e.g. Ref. [7]. The method is based on the possibility to enlarging the space of fields in the quantized theory by auxiliary fields through a supercurvature of a generalized superconnection. The auxiliary fields are required to achieve the off-shell nilpotency of the BRST operator. The minimal set of auxiliary fields is given after having imposed constraints on the generalized supercurvature in which the consistency with the Bianchi identities is guaranteed. We note that the same scheme is applied for the simple supergravity where the classical gauge algebra is open [8].

It is also known that the introduction of auxiliary fields in the quantized BF theories guarantees the off-shell closedness of the superalgebra of the Wess-Zumino type containing besides the BRST symmetry the vector supersymmetry (VSUSY) [9], which was already observed in BF theories [10]. We remark that in the generalized connection formalism as developed in Ref. [9] (see also Ref. [11]) the VSUSY is only determined after having built the quantum action.

Recently, the off-shell BRST-VSUSY superalgebra of BF theories in dimension two [12] and four [13] is considered in terms of superfields defined on the so-called BRST-VSUSY superspace. In Ref. [13], the latter is obtained by extending the BRST superspace with four anticommuting coordinates. In this analysis, the off-shell BRST-VSUSY invariant quantum action is obtained via a twisting process from supersymmetric theories.

Furthermore, the off-shell superalgebra of twisted N=2 superYM theories is constructed by the superconnection formalism of BRST-VSUSY superspace in two and four dimensions [14].

What is still lacking is the construction, in a natural way, of the BRST-VSUSY superalgebra, for D=4 BF theories, in terms of a superconnection defined on the BRST-VSUSY superspace. Inspired mainly by the results obtained in Ref. [6], we would like to define superconnections on the BRST-VSUSY superspace, which allow one to derive ab initio the off-shell BRST-VSUSY superalgebra by using the structure equations and the Bianchi identities. We mention

that, contrary to what happens in the BRST superspace [6], the BRST-VSUSY superspace also leads to the introduction of all auxiliary fields which implement the BRST-VSUSY exactness of the full quantum action. The latter can be obtained from a superaction generalizing that constructed in Ref. [6].

Let us consider the (4,2+4)-dimensional BRST-VSUSY superspace with local coordinates $(z^M, \theta^\mu) = (x^\mu, \theta^\alpha, \theta^\mu)$. Following the guideline of Ref.[6] we introduce over this superspace an even 1-form superconnection ω and an even 2-form generalized superconnection ϕ . By acting the exterior covariant superdifferential D on ω and ϕ , we obtain the supercurvature Ω (even 2-form) and the generalized supercurvature Φ (even 3-form), respectively, satisfying the structure equations

$$\Omega = D\omega = d\omega + \frac{1}{2} [\omega, \omega] , \quad (1)$$

$$\Phi = D\phi = d\phi + [\omega, \phi] , \quad (2)$$

and the Bianchi identities

$$D\Omega = 0 , \quad (3)$$

$$D\Phi = [\Omega, \phi] , \quad (4)$$

where d is the exterior superdifferential and $[\cdot, \cdot]$ the graded Lie bracket. The local expressions of the superconnections and supercurvatures can be written as

$$\omega = dz^M \omega_M + d\theta^\mu \omega'_\mu , \quad (5)$$

$$\phi = \frac{1}{2} \{ dz^N dz^M \phi_{MN} + dz^M d\theta^\mu \phi'_{\mu M} + d\theta^\nu d\theta^\mu \phi''_{\mu\nu} \} , \quad (6)$$

$$\Omega = \frac{1}{2} \{ dz^N dz^M \Omega_{MN} + dz^M d\theta^\mu \Omega'_{\mu M} + d\theta^\nu d\theta^\mu \Omega''_{\mu\nu} \} , \quad (7)$$

$$\begin{aligned} \Phi = & \frac{1}{3!} \{ dz^R dz^N dz^M \Phi_{MNR} + dz^N dz^M d\theta^\mu \Phi'_{\mu MN} + \\ & dz^M d\theta^\nu d\theta^\mu \Phi''_{\mu\nu M} + d\theta^\rho d\theta^\nu d\theta^\mu \Phi'''_{\mu\nu\rho} \} . \end{aligned} \quad (8)$$

We recall that the components of the superconnections and supercurvatures are gauge Lie algebra valued superfields. The superfields components of the supercurvatures are determined by the structure equations. We have

$$\Omega_{MN} = \partial_M \omega_N - (-1)^{mn} \partial_N \omega_M + [\omega_M, \omega_N] , \quad (9)$$

$$\Omega'_{\mu M} = \partial'_\mu \omega_M - (-1)^m \partial_M \omega'_\mu + [\omega'_\mu, \omega_M] , \quad (10)$$

$$\Omega''_{\mu\nu} = \partial'_\mu \omega'_\nu + \partial'_\nu \omega'_\mu + [\omega'_\mu, \omega'_\nu] , \quad (11)$$

$$\Phi_{MNR} = D_M \phi_{NR} + (-1)^{m(n+r)} D_N \phi_{RM} + (-1)^{r(m+n)} D_R \phi_{MN} , \quad (12)$$

$$\Phi'_{\mu MN} = D'_\mu \phi_{MN} - (-1)^m D_M \phi'_{\mu N} + (-1)^{n(m+1)} D_N \phi'_{\mu M} , \quad (13)$$

$$\Phi''_{\mu\nu M} = D'_\mu \phi'_{\nu M} + D'_\nu \phi'_{\mu M} + D_M \phi''_{\mu\nu} , \quad (14)$$

$$\Phi'''_{\mu\nu\rho} = D'_\mu \phi''_{\nu\rho} + D'_\nu \phi''_{\mu\rho} + D'_\rho \phi''_{\mu\nu} , \quad (15)$$

where $m = |z^M|$ is the Grassmann degree of z^M , $D_M = \partial_M + [\omega_M, \cdot]$, $D'_\mu = \partial'_\mu + [\omega'_\mu, \cdot]$ and $\partial'_\mu = \frac{\partial}{\partial \theta^\mu}$. Similarly, the Bianchi identities (3) and (4) become

$$D_M \Omega_{NR} + (-1)^{m(n+r)} D_N \Omega_{RM} + (-1)^{r(m+n)} D_R \Omega_{MN} = 0 , \quad (16)$$

$$D'_\mu \Omega_{MN} - (-1)^m D_M \Omega'_{\mu N} + (-1)^{n(m+1)} D_N \Omega'_{\mu M} = 0 , \quad (17)$$

$$D_M \Omega''_{\mu\nu} + D'_\mu \Omega'_{\nu M} + D'_\nu \Omega'_{\mu M} = 0 , \quad (18)$$

$$D'_\rho \Omega''_{\mu\nu} + D'_\mu \Omega''_{\nu\rho} + D'_\nu \Omega''_{\mu\rho} = 0 , \quad (19)$$

$$D_M \Phi_{NRS} - (-1)^{s(m+n+r)} D_S \Phi_{MNR} + (-1)^{(m+n)(r+s)} D_R \Phi_{SMN} -$$

$$\begin{aligned}
(-1)^{m(n+r+s)} D_N \Phi_{RSM} &= [\Omega_{MN}, \phi_{RS}] + (-1)^{s(n+r)} [\Omega_{MS}, \phi_{NR}] - \\
&(-1)^{nr} [\Omega_{MR}, \phi_{NS}] - (-1)^{m(n+s)+sr} [\Omega_{NS}, \phi_{MR}] + \\
&(-1)^{(m+n)(r+s)} [\Omega_{RS}, \phi_{MN}] + (-1)^{m(n+r)} [\Omega_{NR}, \phi_{MS}] , \tag{20}
\end{aligned}$$

$$\begin{aligned}
D_M \Phi'_{\mu NR} - (-1)^{m(n+r+1)} D'_\mu \Phi_{NRM} + (-1)^{r(m+n+1)+m} D_R \Phi'_{\mu MN} + \\
(-1)^{m(n+r+1)+n} D_N \Phi'_{\mu RM} = -(-1)^n [\Omega_{MN}, \phi'_{\mu R}] - (-1)^{r(m+n)+m} [\Omega'_{\mu R}, \phi_{MN}] \\
- (-1)^m [\Omega'_{\mu M}, \phi_{NR}] + (-1)^{m(n+1)} [\Omega'_{\mu N}, \phi_{MR}] - \\
(-1)^{(r+n)(m+1)+m} [\Omega_{NR}, \phi'_{\mu M}] + (-1)^{r(n+1)} [\Omega_{MR}, \phi'_{\mu N}] , \tag{21}
\end{aligned}$$

$$\begin{aligned}
D_M \Phi''_{\nu \mu N} + D'_\mu \Phi'_{\nu MN} + D'_\nu \Phi'_{\mu MN} - (-1)^{mn} D_N \Phi''_{\nu \mu M} = \\
[\Omega_{MN}, \phi''_{\nu \mu}] - (-1)^m [\Omega'_{\nu M}, \phi'_{\mu N}] + [\Omega''_{\nu \mu}, \phi_{MN}] + (-1)^{n(m+1)} [\Omega'_{\mu N}, \phi'_{\nu M}] - \\
(-1)^m [\Omega'_{\mu M}, \phi'_{\nu N}] + (-1)^{n(m+1)} [\Omega'_{\nu N}, \phi'_{\mu M}] , \tag{22}
\end{aligned}$$

$$\begin{aligned}
(-1)^m D_M \Phi'''_{\rho \nu \mu} - D'_\mu \Phi''_{\rho \nu M} - D'_\nu \Phi''_{\mu \rho M} - D'_\rho \Phi''_{\nu \mu M} = -[\Omega''_{\nu \mu}, \phi'_{\rho M}] - [\Omega''_{\rho \nu}, \phi'_{\mu M}] \\
- [\Omega'_{\rho M}, \phi''_{\nu \mu}] - [\Omega'_{\mu M}, \phi''_{\rho \nu}] - [\Omega'_{\nu M}, \phi''_{\rho \mu}] - [\Omega''_{\rho \mu}, \phi'_{\nu M}] , \tag{23}
\end{aligned}$$

$$\begin{aligned}
D'_\sigma \Phi'''_{\rho \nu \mu} + D'_\mu \Phi'''_{\sigma \rho \nu} + D'_\nu \Phi'''_{\mu \sigma \rho} + D'_\rho \Phi'''_{\nu \mu \sigma} = [\Omega''_{\sigma \nu}, \phi''_{\rho \mu}] + [\Omega''_{\rho \mu}, \phi''_{\sigma \nu}] \\
[\Omega''_{\sigma \rho}, \phi''_{\nu \mu}] + [\Omega''_{\mu \sigma}, \phi''_{\rho \nu}] + [\Omega''_{\nu \mu}, \phi''_{\sigma \rho}] + [\Omega''_{\rho \nu}, \phi''_{\mu \sigma}] . \tag{24}
\end{aligned}$$

To translate the above equations into equations determining the off-shell BRST-VSUSY superalgebra of D=4 BF theories, we should

interpret the fields occuring in such theories geometrically. We note that the physical fields and the auxiliary fields are introduced through the superconnections and the supercurvatures, respectively. The auxiliary fields are required for the construction of the off-shell structure of the theory. To this end, we will be only interested to the superfields ω_M and ϕ_{MN} which give the physical fields as constructed in Ref.[6], i.e. we can put

$$\omega'_\mu = 0, \quad \phi'_{\mu M} = 0, \quad \phi''_{\mu\nu} = 0. \quad (25)$$

At this point, we remark that the lowest components $\omega_\mu |$, $\omega_1 |$, $\omega_2 |$, $\partial_1 \omega_2 |$ and $\Omega_{\mu\nu} |$ describe as usual the gauge potential A_μ^0 , the ghost A_0^1 , the anti-ghost A_0^{-1} , the Stueckelberg field b_0^0 and the YM curvature $F_{\mu\nu}^0$, respectively. The symbol " $|$ " indicates that the quantity is evaluated at $\theta^\alpha = 0$ and $\theta^\mu = 0$. Here and in what follows, we have used the convention that a field X with Lorentz degree p and ghost number q is denoted $X_{\mu_1 \dots \mu_p}^q$ (or X_p^q). Besides the A_μ^0 system of fields related to the YM symmetry, i.e. $\{A_\mu^0, A_0^1, A_0^{-1}, b_0^0\}$, there exists the system of fields related to the reducible symmetry and associated with the rank-two tensor gauge field $B_{\mu\nu}^0$. This is identified as follows: $B_{\mu\nu}^0 = \phi_{\mu\nu} |$, $B_\mu^1 = \phi_{\mu 1} |$ is the ghost associated with $B_{\mu\nu}^0$, $B_\mu^{-1} = \phi_{\mu 2} |$ is the anti-ghost of B_μ^1 , $\pi_\mu^0 = \partial_1 \phi_{\mu 2} |$ is the associated Stueckelberg field, $B_0^2 = \phi_{11}/2 |$ is the ghost for the ghost B_μ^1 , $B_0^{-2} = \phi_{22}/2 |$ is the anti-ghost of B_0^2 , $\pi_0^{-1} = \partial_1 \phi_{22} |$ is the associated Stueckelberg field, $K_{\mu\nu\rho}^0 = \Phi_{\mu\nu\rho} |$ is the curvature of $B_{\mu\nu}^0$ and $(B_0^0 = \phi_{12} |$, $\pi_0^1 = \partial_1 \phi_{12} |)$ is a pair of fields which takes into account a further degeneracy associated with π_μ^0 .

The next step is to search for constraints to the supercurvatures in which the consistency with the Bianchi identities is ensured. This requirement guarantees then the off-shell closedness of the BRST-VSUSY superalgebra thanks to the structure equations and the Bianchi identities. First, we note that the BRST transformations are obtained by imposing as usual the following constraints

$$\Omega_{\mu\alpha} = \Omega_{\alpha\beta} = 0, \quad \Phi_{\mu\nu 2} = \Phi_{\mu 22} = \Phi_{\mu 12} = \Phi_{222} = 0. \quad (26)$$

It is easy to see through an analysis of the Bianchi identities (16) and (20) that the constraints (26) fulfill the consistency with these equations. However, it is worth noting that the remaining supercurvature components $\Phi_{\mu\nu 1}$, $\Phi_{1\mu 1}$, Φ_{111} permit us to introduce auxiliary fields defined by

$$\Phi_{\mu\nu 1} |= E_{\mu\nu}^1 , \quad \frac{1}{2} \Phi_{1\mu 1} |= E_{\mu}^2 , \quad \frac{1}{6} \Phi_{111} |= E_0^3 , \quad (27)$$

which are required for the construction of the off-shell nilpotent BRST transformations (see Ref.[6]). The superfields $\Phi_{\mu\nu 1}$, $\Phi_{1\mu 1}$, Φ_{111} can be expanded in power series of θ^μ as follows

$$\begin{aligned} \Phi_{\mu\nu 1} &= E_{\mu\nu}^1(z) + \theta^\rho E_{\mu\nu\rho}^0(z) + \frac{1}{2!} \theta^\sigma \theta^\rho E_{\mu\nu\rho\sigma}^{-1}(z), \\ \frac{1}{2} \Phi_{1\mu 1} &= E_{\mu}^2(z) + \theta^\nu E_{\mu\nu}^1(z) + \frac{1}{2!} \theta^\rho \theta^\nu E_{\mu\nu\rho}^0(z) + \frac{1}{3!} \theta^\sigma \theta^\rho \theta^\nu E_{\mu\nu\rho\sigma}^{-1}(z), \\ \frac{1}{6} \Phi_{111} &= E_0^3(z) + \theta^\mu E_{\mu}^2(z) + \frac{1}{2!} \theta^\nu \theta^\mu E_{\mu\nu}^1(z) + \frac{1}{3!} \theta^\rho \theta^\nu \theta^\mu E_{\mu\nu\rho}^0(z) + \\ &\quad \frac{1}{4!} \theta^\sigma \theta^\rho \theta^\nu \theta^\mu E_{\mu\nu\rho\sigma}^{-1}(z), \end{aligned} \quad (28)$$

where the set of auxiliary fields $\{E_{\mu\nu}^1, E_{\mu}^2, E_0^3, E_{\mu\nu\rho}^0, E_{\mu\nu\rho\sigma}^{-1}\}$, which is obtained by evaluating the superfields components in the development (28) at $\theta^\alpha = 0$, are needed for the construction of the off-shell structure of D=4 BF theories.

Obviously, from (10), (11), (13), (14) and (15) and in view of (25), it follows that

$$\Omega''_{\mu\nu} = 0 , \quad \Phi''_{\mu\nu M} = \Phi'''_{\mu\nu\rho} = 0 , \quad (29)$$

$$\Omega'_{\mu M} = \partial'_\mu \omega_M , \quad (30)$$

$$\Phi'_{\mu MN} = \partial'_\mu \phi_{MN} , \quad (31)$$

which satisfy automatically the consistency of the Bianchi identities (18), (19), (22), (23) and (24).

Moreover, through an analysis of the Bianchi identity (17), we deduce the following solutions

$$\Omega'_{\mu 1} = \omega_\mu , \quad \Omega'_{\mu 2} = 0. \quad (32)$$

These permit us to introduce an auxiliary field given by

$$\Omega'_{\mu\nu} | = H_{\mu\nu}^{-1} , \quad (33)$$

which is required for the construction of off-shell BRST-VSUSY transformations. The superfield $\Omega'_{\mu\nu}$ can be expanded in power series of θ^μ as follows

$$\Omega'_{\mu\nu} = H_{\mu\nu}^{-1}(z) + \theta^\rho H_{\mu\nu\rho}^{-2}(z) + \frac{1}{2!} \theta^\rho \theta^\sigma H_{\mu\nu\sigma\rho}^{-3}(z) . \quad (34)$$

The set of auxiliary fields $\{H_{\mu\nu}^{-1}, H_{\rho\mu\nu}^{-2}, H_{\sigma\rho\mu\nu}^{-3}\}$, which is obtained by evaluating the superfields components in the development (34) at $\theta^\alpha = 0$, is also needed for the construction of the off-shell structure of D=4 BF theories.

Turning now to the situation of the superfields $\Phi'_{\mu MN}$, a similar analysis of the Bianchi identity (21) leads to the following solutions

$$\Phi'_{\mu\nu 1} = \phi_{\mu\nu} , \quad \frac{1}{2} \Phi'_{\mu 11} = \phi_{\mu 1} , \quad \Phi'_{\mu 22} = \Phi'_{\mu 12} = \Phi'_{\mu 21} = 0 . \quad (35)$$

Again, the remaining superfields $\Phi'_{\mu\nu\rho}$ and $\Phi'_{\mu\nu 2}$ as given in (31) ensure automatically the consistency of the Bianchi identities and in order to find exactly the off-shell BRST-VSUSY transformations, we can choose

$$\Phi'_{\mu\nu\rho} | = \varepsilon_{\mu\nu\rho\sigma} \partial^\sigma A_0^{-1} , \quad \Phi'_{\mu\nu 2} | = -\eta_{\mu\nu} B_0^{-2} , \quad (36)$$

where $\varepsilon_{\mu\nu\rho\sigma}$ is the antisymmetric Levi-civita tensor and $\eta_{\mu\nu}$ is the flat Minkowskian metric in 4-dimensions.

However, in order to obtain the off-shell BRST-VSUSY transformations of all the fields, we note that the BRST operator is related as usual to the partial derivative ∂_1 [6], whereas the VSUSY operator can be related to the differential operator $D_{\theta^\mu} = \partial'_\mu + \theta \partial_\mu$. So, we can make the following substitution

$$\begin{aligned} \Omega'_{\mu M} &\longrightarrow \Lambda_{\mu M} = \Omega'_{\mu M} + \theta \partial_\mu \omega_M , \\ \Phi'_{\mu MN} &\longrightarrow \Pi_{\mu MN} = \Phi'_{\mu MN} + \theta \partial_\mu \phi_{MN} , \end{aligned} \quad (37)$$

where, we have remplaced ∂'_μ by D_{θ^μ} in the superfield components $\Omega'_{\mu M}$ and $\Phi'_{\mu MN}$.

Then, we realize the following identifications

$$Q(\psi|) = (\partial_1\psi)|, \quad (38)$$

$$Q_\mu(\psi|) = (D_{\theta^\mu}\psi)|, \quad (39)$$

where Q and Q_μ are interpreted as the BRST and VSUSY operator for D=4 BF theories, respectively, and ψ is any superfield with the replacements (37).

Furthermore, from the Bianchi identity (21) and in view of (37), we deduce that

$$\begin{aligned} E_{\mu\nu\rho}^0 &= -\varepsilon_{\mu\nu\rho\sigma}\partial^\sigma b_0^0 - \varepsilon_{\mu\nu\rho\sigma}[A_0^1, \partial^\sigma A_0^{-1}] - K_{\mu\nu\rho}^0 + [H_{\mu\nu}^{-1}, B_\rho^1] - [H_{\mu\rho}^{-1}, B_\nu^1], \\ E_{\lambda\mu\nu\rho}^{-1} &= -[H_{\lambda\mu}^{-1}, B_{\nu\rho}^0] - [H_{\lambda\nu}^{-1}, B_{\rho\mu}^0] - [H_{\lambda\rho}^{-1}, B_{\mu\nu}^0] - [H_{\mu\nu}^{-1}, B_{\lambda\rho}^0] + \\ &\quad [H_{\mu\rho}^{-1}, B_{\lambda\nu}^0] + [H_{\mu\nu\lambda}^{-2}, B_\rho^1] - [H_{\mu\rho\lambda}^{-2}, B_\nu^1]. \end{aligned} \quad (40)$$

In the end, by inserting (26), (28), (29), (32), (34), (35) and (36) in the structure equations and the Bianchi identities, and by using the identification (38), we deduce the off-shell BRST transformations of the auxiliary fields and the original fields as given in Ref. [6] (see also Ref. [9])

$$QA_0^1 = -\frac{1}{2}[A_0^1, A_0^1], \quad QA_\mu^0 = D_\mu A_0^1,$$

$$QB_{\mu\nu}^0 = -(D_\mu B_\nu^1 - D_\nu B_\mu^1) - [A_0^1, B_{\mu\nu}^0] + E_{\mu\nu}^1,$$

$$QB_\mu^1 = D_\mu B_0^2 - [A_0^1, B_\mu^1] + E_\mu^2, \quad QB_0^2 = -[A_0^1, B_0^2] + E_0^3,$$

$$QH_{\mu\nu}^{-1} = F_{\mu\nu} - [A_0^1, H_{\mu\nu}^{-1}],$$

$$QH_{\mu\nu\rho}^{-2} = -\sum_{(\mu\nu\rho)} D_\mu H_{\nu\rho}^{-1} - [A_0^1, H_{\mu\nu\rho}^{-2}],$$

$$QH_{\mu\nu\rho\sigma}^{-3} = \sum_{(\mu\nu\rho\sigma)} D_\mu H_{\nu\rho\sigma}^{-2} - [A_0^1, H_{\mu\nu\rho\sigma}^{-3}] + \frac{1}{2} \sum_{(\mu\nu\rho\sigma)} [H_{\mu\nu}^{-1}, H_{\rho\sigma}^{-1}] ,$$

$$QE_{\mu\nu}^1 = \sum_{(\mu\nu)} D_\mu E_\nu^2 - [A_0^1, E_{\mu\nu}^1] + [F_{\mu\nu}^0, B_0^2] ,$$

$$QE_\mu^2 = D_\mu E_0^2 - [A_0^1, E_\mu^2] , \quad QE_0^3 = -[A_0^1, E_0^3] ,$$

$$QA_0^{-1} = b_0^0 , \quad Qb_0^0 = 0 ,$$

$$QB_\mu^{-1} = \pi_\mu^0 , \quad Q\pi_\mu^0 = 0 ,$$

$$QB_0^{-2} = \pi_0^{-1} , \quad Q\pi_0^{-1} = 0 ,$$

$$QB_0^0 = \pi_0^1 , \quad Q\pi_0^1 = 0 , \quad (41)$$

where $\sum_{(MN..)}$ means a cyclic sum over M, N,... .

Then, by using (39), we obtain the following VSUSY transformations as given in Ref.[9].

$$Q_\mu A_0^1 = A_\mu^0 , \quad Q_\mu A_\nu^0 = H_{\mu\nu}^{-1} , \quad Q_\mu H_{\nu\rho}^{-1} = H_{\nu\rho\mu}^{-2} ,$$

$$Q_\mu H_{\nu\rho\sigma}^{-2} = H_{\nu\rho\mu\sigma}^{-3} , \quad Q_\mu H_{\nu\rho\sigma\tau}^{-3} = 0 ,$$

$$Q_\mu B_0^2 = B_\mu^1 , \quad Q_\mu B_\nu^1 = B_{\mu\nu}^0 , \quad Q_\mu B_{\nu\rho}^0 = \varepsilon_{\mu\nu\rho\sigma} \partial^\sigma A_0^{-1} ,$$

$$Q_\mu E_0^3 = E_\mu^2 , \quad Q_\mu E_\nu^2 = E_{\nu\mu}^1 , \quad Q_\mu E_{\nu\rho}^1 = E_{\nu\rho\mu}^0 ,$$

$$Q_\mu E_{\nu\rho\sigma}^0 = E_{\nu\rho\mu\sigma}^{-1} , \quad Q_\mu E_{\nu\rho\sigma\lambda}^{-1} = 0 ,$$

$$Q_\mu A_0^{-1} = 0 , \quad Q_\mu b_0^0 = \partial_\mu A_0^{-1} ,$$

$$\begin{aligned}
Q_\mu B_\nu^{-1} &= -\eta_{\mu\nu} B_0^{-2} \quad , \quad Q_\mu \pi_\nu^0 = \partial_\mu B_\nu^{-1} + \eta_{\mu\nu} \pi_0^{-1} \quad , \\
Q_\mu B_0^{-2} &= 0 \quad , \quad Q_\mu \pi_0^{-1} = \partial_\mu B_0^{-2} \quad , \\
Q_\mu B_0^0 &= 0 \quad , \quad Q_\mu \pi_0^1 = \partial_\mu B_0^0 \quad .
\end{aligned} \tag{42}$$

It is easy to see that the BRST-VSUSY superalgebra

$$\{Q, Q\} = 0 \quad , \quad \{Q_\mu, Q_\nu\} = 0 \quad , \quad \{Q, Q_\mu\} = \partial_\mu \tag{43}$$

is automatically satisfied off-shell, thanks to the structure equations and the Bianchi identities.

Finally, in order to arrive at the off-shell BRST-VSUSY quantum action of D=4 BF theories, let us recall that the gauge fixing action of D=4 BF theories in the BRST superspace formalism, is constructed from a gauge fixing superaction given by [6]

$$\begin{aligned}
S_{sgf} &= \partial_1 \omega_2 \partial^\mu \omega_\mu - \omega_2 \partial^\mu \partial_1 \omega_\mu + \partial_1 \phi_2^\mu \partial^\nu \phi_{\nu\mu} - \phi_2^\mu \partial^\nu \partial_1 \phi_{\nu\mu} + \partial_1 \phi_1^\mu \partial_\mu \phi_{22} - \\
&\quad \phi_1^\mu \partial_\mu \partial_1 \phi_{22} + \partial_1 \phi_2^\mu \partial_\mu \phi_{12} - \phi_2^\mu \partial_\mu \partial_1 \phi_{12} + \partial_1 \phi_{22} \partial_1 \phi_{12} \\
&= \partial_1 \left[\omega_2 \partial^\mu \omega_\mu + \phi_2^\mu \partial^\nu \phi_{\nu\mu} + \phi_1^\mu \partial_\mu \phi_{22} + \phi_2^\mu \partial_\mu \phi_{12} + \phi_{22} \partial_1 \phi_{12} \right] .
\end{aligned} \tag{44}$$

In the BRST-VSUSY superspace formalism, (44) can be written, up to a total divergence, as

$$S_{sgf} = -\partial_1 D_{\theta^\mu} (\omega_2 \partial^\mu \omega_1 + \phi_2^\mu \partial^\nu \phi_{\nu 1} + \phi_2^\mu \partial_1 \phi_{12}) . \tag{45}$$

Furthermore, in order to find the classical action, we define the extended classical superaction as follows

$$S_s^0 = \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} \partial_1 D_{\theta^\sigma} (\Omega'_{\mu\nu} \phi_{\rho 1} + \frac{1}{6} \partial_\rho' \Omega'_{\mu\nu} \phi_{11}) . \tag{46}$$

It is easy to see that the full quantum action S_q defined by

$$S_q = (S_s^0 + S_{sgf}) \mid \tag{47}$$

is BRST-VSUSY exact. It can be also written in the BRST exact form leading to the same quantum action with the Landau gauge obtained in the context of the generalized connection formalism [9].

In conclusion, working with the same spirit as in Ref.[6], we have performed the quantization of D=4 BF theories by using the superconnection formalism. In this analysis, the A_μ^0 and $B_{\mu\nu}^0$ systems of fields are described as usual through even superconnections over a BRST-VSUSY superspace. Using the exterior covariant superdifferential on the superconnections gives even supercurvatures, which lead to the determination of the off-shell BRST-VSUSY transformations. The off-shell closedness is realized by introducing through the supercurvatures two minimal sets of auxiliary fields required for the consistency of the BRST-VSUSY superspace geometry. Finally, in order to find the full quantum action, we have built the superaction generalizing that in Ref.[6]. This results in the construction of the off-shell BRST-VSUSY exact quantum action of D=4 BF theories.

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